$$Sp\left(\frac{\partial^2 f}{\partial p^2}\cdot\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 f}{\partial x\,\partial p}\right)$$

$f = p_j f_0 \qquad \frac{\partial f}{\partial p_k} = p_j \frac{\partial f_0}{\partial p_k} + \delta_{jk} f_0$

$$\frac{\partial^2 f}{\partial p_i \partial p_k} = p_j \frac{\partial^2 f_0}{\partial p_i \partial p_k} + \delta_{ij}\frac{\partial f_0}{\partial p_k} + \delta_{jk}\cdot\frac{\partial f_0}{\partial p_i}$$

$$Sp\left(\frac{\partial^2 f}{\partial p^2}\cdot\frac{\partial^2 s}{\partial x^2}\right) = \frac{\partial^2 f}{\partial p_i \partial p_k}\cdot\frac{\partial^2 s}{\partial x_k \partial x_i} = p_j \frac{\partial^2 f_0}{\partial p_i \partial p_k}\cdot$$

$$\cdot\frac{\partial^2 s}{\partial x_k \partial x_i} + \delta_{ij}\frac{\partial f_0}{\partial p_k}\cdot\frac{\partial^2 s}{\partial x_k \partial x_i} + \delta_{jk}\frac{\partial f_0}{\partial p_i}\frac{\partial^2 s}{\partial x_k \partial x_i} =$$

$$= p_j\, Sp\left(\frac{\partial^2 f_0}{\partial p^2}\,\frac{\partial^2 s}{\partial x^2}\right) + 2\left(\frac{\partial f_0}{\partial p}, \frac{\partial^2 s}{\partial x\,\partial x_j}\right)$$

$$\frac{\partial^2 f}{\partial x_k \partial p_k} = p_j \frac{\partial^2 f_0}{\partial p_k \partial x_k} + \delta_{jk}\frac{\partial f_0}{\partial x_k}$$

$$\underset{\text{след}}{Sp}\,\frac{\partial^2 f}{\partial x\,\partial p} = p_j\; Sp\,\frac{\partial^2 f_0}{\partial p\,\partial x} + \frac{\partial f_0}{\partial x_j}.$$

Итак, всё доказали.

Применим лемму, чтобы найти реш. ур-ие $\hat H\psi = E\psi$ при $h \longrightarrow 0$, где $\psi = \psi(x_1,\dots,x_n)$.

В случае: $\psi_m = p_m\left(\frac{x}{\sqrt h}\right) e^{-\frac{\omega x^2}{2h}}$

$$\hat H = -\frac{h^2}{2}\frac{d^2}{dx^2} + \frac{\omega^2 x^2}{2}$$

$$E_m = h\omega\left(m+\tfrac12\right);\; E_m \xrightarrow[h\to 0]{} 0$$

Если рассм-ть: $H = \frac{p^2}{2} + \frac{\omega^2 x^2}{2}$

$H = 0 \qquad p = x = 0$ — полож. равнов.